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## **Inflation-Conservatism and Monetary- Fiscal Policy Interactions**

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# Inflation-Conservatism and Monetary-Fiscal Policy Interactions\*

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## Abstract

This paper investigates the stabilization bias that arises in a model of monetary and fiscal policy stabilization of the economy, when assuming that the monetary authority put higher weight on inflation stabilization than society. We demonstrate that inflation-conservatism unambiguously leads to social welfare losses if the fiscal authority acts strategically. Although the precise form of monetary-fiscal interactions depends on the choice of fiscal instrument and on the level of steady state debt, the assessment of gains is robust to these assumptions. We also study how the outcome of stabilization depends on the leadership structure. We develop an algorithm that computes leadership equilibria as well as in much wider spectrum of problems with strategic agents.

Key Words: Monetary and Fiscal Policy, Policy Delegation, Discretion, Leadership Equilibria

JEL References: E31, E52, E58, E61, C61

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\*This paper was completed when Tatiana Kirsanova was a Houblon-Norman/George Research Fellow at the Bank of England. It represents the views and analysis of the authors and should not be thought to represent those of the Bank of England or Monetary Policy Committee members. Any errors remain ours.

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# 1 Introduction

There is a well understood policy proposal that the agency charged with determining monetary policy, usually the central bank, should be ‘inflation-conservative’, by which it is meant that it should have a higher weight on inflation stabilization relative to other objectives of policy than is socially optimal. The logic runs as follows. Suppose the policymaker must act under discretion. Also suppose, for some reason (and we nearly always assume some political economy issues here), it wants to have lower unemployment or equivalently higher output. Such a policymaker will generate an excessively high equilibrium *rate* of inflation, a *level bias*. This is the now-textbook Barro and Gordon (1983) model. One possible resolution of this problem is to make the central bank both independent and inflation-conservative, as famously shown by Rogoff (1985). This removes some of the incentive to lower unemployment, and hence reduces the inflation bias. This analysis, and its myriad extensions, has been very influential: most central banks in developed countries are now operationally and/or target independent. Indeed, central banks are often bestowed with the soubriquet ‘conservative’ in the popular press, even if mitigating the inflationary bias is not quite what they have in mind.

There seems little doubt that major central banks do not pursue excessive output targets and none of them aims to stabilize inflation at too high a level. Contemporary policymakers demonstrate considerable agreement about what the *targets* of policy should be. Nonetheless, in *dynamic* models a *stabilization bias* might still arise. The *volatility* of economic variables is necessarily higher if a policymaker acts under discretion (see e.g. Currie and Levine (1993)). Once again, delegating policy to a conservative central bank could still lead to an improved outcome, as Svensson (1987) demonstrates. Clarida et al. (1999) show that an inflation-conservative monetary authority that acts under discretion can achieve the same level of welfare as under the optimal precommitment-to-rules policy. In other words, the stabilization bias can be reduced: making the central bank inflation-conservative helps solve the problem of optimal delegation.<sup>1</sup> It seems reasonable to conclude that the conservative-central-bank proposal is a good one, as it deals with both the level and the stabilization bias.

However, simply separating the central bank from the government and making the central bank inflation-conservative does not eliminate the influence of the *government*, who still controls the fiscal instruments of financial management. It may deviate from what is socially optimal and, say, have a lower unemployment target or prefer more stable growth. Even if it remains entirely benevolent it may still behave *strategically*<sup>2</sup> which can affect the resulting equilibrium. Both level and stabilization bias can still arise.

Indeed, Dixit and Lambertini (2003), using a static Barro-Gordon type model, demonstrate that a conflict of interests does make the outcome suboptimal in a model of monetary-fiscal interactions with a conservative central bank and a *benevolent* fiscal authority. The level bias still arises: the levels of inflation and unemployment in equilibrium are both higher than socially

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<sup>1</sup>The optimal delegation is usually described as a possibility to distort targets of a discretionary policymaker so that it can achieve higher social welfare than it would achieve if it were benevolent.

<sup>2</sup>There is little doubt that the fiscal authorities act strategically: An existing empirical literature on monetary-fiscal interactions suggests that fiscal policy does more than simply allowing automatic stabilisers to operate, see e.g. Auerbach (2002) and Favero and Monacelli (2005) who analyze fiscal policy in the US. Moreover, since the onset of European monetary union there are calls for greater fiscal flexibility, although how strategic such authorities should be is not explicitly discussed.

optimal. With an additional policymaker it seems the conservative-central-bank proposal does not work out even in a *static* model.<sup>3</sup> Similarly, the conservative-central-bank solution to an optimal delegation problem in a *dynamic* stochastic environment could also be misleading if potentially strategic play by policymakers is ignored.<sup>4</sup>

In further related research, Adam and Billi (2007) examine the advantages of inflation conservatism using a non-linear dynamic model similar to ours and find that it can be beneficial. However, they mostly study the implications for the steady-state, and limit their analysis of stabilization bias to the case of strategic fiscal leadership and a flexible price version of the model. By contrast, our paper is concerned with the stabilization aspects of the problem and can be seen as a complement to their paper.

Making the model dynamic furthers the analysis on the two accounts. First, we argue that there is now a wide consensus about appropriate level targets for the financial authorities, so the focus of the policy debate is often *how quickly* should they try to achieve these targets, rather than *which targets* to achieve. Second, modelling monetary-fiscal interactions in a dynamic setting allows us to study the effect of the governments solvency constraint. Debt stabilization issues can impose severe restrictions on the stabilization abilities of both authorities (see Leeper (1991)) but it is very difficult to model these restrictions adequately in a static model.

In this paper we explore the importance of both strategic behavior and inflation-conservatism in a dynamic model of a kind widely used in policy analysis. We employ a conventional model with monopolistic competition and sticky prices in the goods markets (as in Woodford (2003), Ch. 6), extended to include fiscal policy and nominal government debt. We allow both authorities to act strategically and non-cooperatively in pursuit of their own objectives. We then delegate monetary policy to an inflation-conservative central bank. We demonstrate that even if the fiscal authority remains benevolent but acts strategically, delegating monetary policy in this way leads to welfare losses. The basic intuition is that if the authorities' objectives do not coincide then one strategic policymaker can offset the policy of the other. We show that although a small degree of conservatism can be harmless, greater conservatism leads to substantial welfare losses.

As in Dixit and Lambertini (2003), the quantitative outcome of the game depends on the leadership structure. We study three possibilities: either the monetary or fiscal authorities lead or they play a Nash game. This analysis is impossible to conduct without developing an appropriate modelling framework. As even moderately complicated dynamic model needs to be solved using numerical methods and these methods are neither readily available nor well articulated for models with rational expectations,<sup>5</sup> we develop relevant solution methods for discretionary equilibria in dynamic linear rational expectations models where we make the role of leadership explicit. Although this is a key contribution of the paper and necessary step in our analysis, we relegate detailed discussion to Appendix A.<sup>6</sup>

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<sup>3</sup>These results were further developed in Lambertini (2006) specifically for the conservative central bank proposal.

<sup>4</sup>For example, in the UK the Bank of England and the Treasury would deny that they 'play games'. However, by 'playing a game' we model the ability of each authority to understand the other's priorities and reaction functions.

<sup>5</sup>de Zeeuw and van der Ploeg (1991) provide an excellent discussion of discrete dynamic games which can be compared with our analysis.

<sup>6</sup>In all that follows we adopt the terminology of dynamic game theory, exemplified in Başar and Olsder (1999), although some of the game is implied rather than explicit. We would argue that much of the recent monetary policy literature constructs consistent equilibria with little regard to the underlying strategic behavior. We feel that a correct, transparent treatment of potential interactions is vital to the understand the resulting policy equilibria.

This paper differs from Dixit and Lambertini (2003), and Adam and Billi (2007) in four important respects. First, we study the stabilization rather than the level bias, and focus on dynamics rather than the steady state. Second, we examine different leadership regimes, and provide suitable algorithms for calculating appropriate equilibria. In so doing we solve rather a general model within a class of dynamic models that are commonly adopted in policymaking. Third, we explicitly account for the effects of potential fiscal insolvency. Finally, we show how the interaction of the two authorities depends on the steady-state level of debt.

Our results are in marked contrast to the received wisdom outlined above. We find that if the fiscal authority is benevolent but acts strategically, then delegating monetary policy to an inflation conservative agency usually *increases* stabilization bias and so reduces social welfare. Any distortion to the social objectives can make the two policymakers ‘fight’ each other in a way that nearly always reduces social welfare for our model. The message is clear: What works well in an economy with a single policymaker may not work at all in an economy with two strategic policymakers. Our assessment of the gains from delegation seem robust to the specification of the model and the choice of fiscal policy instrument. We conduct sensitivity analysis over the model, and show that whilst there are differences in the behavior of fiscal policy if we choose either distortionary taxes or spending, the qualitative results are the same. However, there is an issue about the steady-state level of debt. The low and high debt cases (defined below) have quite different qualitative effects.

The paper is organized as follows. In the next section we present the model, where we discuss the derivation of key equations, and derive social welfare. Section 3 presents the analysis of the two policy regimes: two benevolent policymakers and conservative central bank. Section 4 investigates how our results change if fiscal policy uses tax as an instrument instead or together with spending. Section 5 concludes. An appendix describes the methods used to compute the equilibria.

## 2 The Model

We consider the now-mainstream macro model, discussed in Woodford (2003), slightly modified to take account of the effects of fiscal policy.<sup>7</sup> It is a closed economy with two policymakers, the fiscal and monetary authorities. Fiscal policy is allowed to support monetary policy in stabilization of the economy around the steady state.

### 2.1 Consumers

Our model of the household sector is familiar from Woodford (2003). Our economy is inhabited by a large number of individuals, who specialize in the production of a differentiated good (indexed by  $z$ ), and who spend  $h(z)$  of effort in its production. They consume a basket of goods  $C$ , and derive utility from per capita government consumption  $G$ . Individuals’ maximization problem is

$$\max_{\{C_v, h_v\}_{v=t}^{\infty}} \mathcal{E}_t \sum_{v=t}^{\infty} \beta^{v-t} [u(C_v) + f(G_v) - v(h_v(z))]. \quad (1)$$

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<sup>7</sup>Similar to Kirsanova et al. (2007), among others.

The price of a differentiated good  $z$  is denoted by  $p(z)$ , and the aggregate price level is  $P$ . An individual chooses optimal consumption and work effort to maximize criterion (1) subject to the demand system and the flow budget constraint:

$$P_t C_t + \mathcal{E}_t(Q_{t,t+1} \mathcal{A}_{t+1}) \leq \mathcal{A}_t + (1 - \Upsilon_t)(w_t(z)h_t(z) + \Pi_t(z)) + T, \quad (2)$$

where  $P_t C_t = \int_0^1 p(z)c(z)dz$  is nominal consumption,  $\mathcal{A}_t$  are nominal financial assets of a household,  $\Pi_t$  is profit and  $T$  is a constant lump-sum tax/subsidy. Here  $w$  is the wage rate, and  $\Upsilon_t$  is a tax rate on income.  $Q_{t,t+1}$  is the stochastic discount factor which determines the price in period  $t$  to the individual of being able to carry a state-contingent amount  $\mathcal{A}_{t+1}$  of wealth into period  $t + 1$ . The riskless short term nominal interest rate  $i_t$  has the following representation in terms of the stochastic discount factor:

$$\mathcal{E}_t(Q_{t,t+1}) = \frac{1}{(1 + i_t)}.$$

Each individual consumes the same basket of goods. Goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by  $\epsilon_t > 1$  (which is a stochastic elasticity with mean  $\epsilon^8$ ),  $C_t = \left[ \int_0^1 c_t^{\frac{\epsilon_t-1}{\epsilon_t}}(z) dz \right]^{\frac{\epsilon_t}{\epsilon_t-1}}$ .

We assume that the net present value of individual's future income is bounded. We also assume that the nominal interest rate is positive at all times. These assumptions rule out infinite consumption and allow us to replace the infinite sequence of flow budget constraints of the individual by a single intertemporal constraint,

$$\mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v} C_v P_v \leq \mathcal{A}_t + \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v} \{ (1 - \Upsilon_v)(w_v(z)h_v(z) + \Pi_v(z)) + T \}. \quad (3)$$

The optimization requires that the household exhaust its intertemporal budget constraint and, in addition, the household's wealth accumulation must satisfy the no Ponzi game condition:

$$\lim_{s \rightarrow \infty} \mathcal{E}_t(Q_{t,s} \mathcal{A}_s) = 0. \quad (4)$$

We assume the specific functional form for the utility from consumption component,  $u(C_v, \xi_t) = \frac{C_v^{1-1/\sigma}}{1-1/\sigma}$ . Household optimization leads to the following dynamic relationship for aggregate consumption:

$$C_t = \mathcal{E}_t \left( \left( \frac{1}{\beta} \frac{P_{t+1}}{P_t} Q_{t,t+1} \right)^\sigma C_{t+1} \right). \quad (5)$$

Additionally, aggregate (nominal) asset accumulation is given by

$$\mathcal{A}_{t+1} = (1 + i_t)(\mathcal{A}_t + (1 - \Upsilon_t)(W_t N_t + \Pi_t) - P_t C_t - T), \quad (6)$$

where  $W_t$  and  $N_t$  are aggregate wage and employment.

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<sup>8</sup>We make this parameter stochastic to allow us to generate shocks to the mark-up of firms.

We linearize equation (5) around the steady state (here and everywhere below for each variable  $X_t$  with steady state value  $X$ , we use the notation  $\hat{X}_t = \ln(X_t/X)$ ). Equation (5) leads to the following Euler equation (intertemporal IS curve):

$$\hat{C}_t = \mathcal{E}_t \hat{C}_{t+1} - \sigma (\hat{i}_t - \mathcal{E}_t \hat{\pi}_{t+1}). \quad (7)$$

Inflation is  $\pi_t = \frac{P_t}{P_{t-1}} - 1$  and we assume steady state inflation is zero.

## 2.2 Price Setting

Price setting is based on Calvo contracting as set out in Woodford (2003). Each period agents recalculate their prices with probability  $1 - \gamma$ . If prices are not recalculated (with probability  $\gamma$ ), they remain fixed. Following Woodford (2003) and allowing for government consumption terms in the utility function, we can derive the following Phillips curve for our economy<sup>9</sup>:

$$\hat{\pi}_t = \beta \mathcal{E}_t \hat{\pi}_{t+1} + \frac{(1 - \gamma\beta)(1 - \gamma)\psi}{\gamma(\psi + \epsilon)} \hat{s}_t, \quad (8)$$

where marginal cost is

$$\hat{s}_t = \frac{1}{\psi} \hat{Y}_t + \frac{1}{\sigma} \hat{C}_t + \frac{\tau}{(1 - \tau)} \hat{\Upsilon}_t + \hat{\eta}_t.$$

The shock  $\hat{\eta}_t$  is a mark-up shock. Here  $\psi = v_y/v_{yy}y$ .

Under flexible prices and in the steady state the real wage is always equal to the monopolistic mark-up  $\mu_t = -(1 - \epsilon_t)/\epsilon_t$ . Optimization by consumers then implies:

$$\frac{\mu^w}{\mu_t} = \frac{v_y(y_t^n(z))}{Q_t (1 - \hat{\Upsilon}_t^n) u_C(C_t^n)}, \quad (9)$$

where superscript  $n$  denotes natural levels (see Woodford (2003)), and  $\mu^w$  is a steady state employment subsidy which we discuss below. Linearization of (9) yields

$$\hat{Y}_t^n \frac{1}{\psi} + \hat{C}_t^n \frac{1}{\sigma} + \frac{\tau}{(1 - \tau)} \hat{\Upsilon}_t^n = 0. \quad (10)$$

## 2.3 Fiscal Constraint

The government buys goods ( $G_t$ ), taxes income (with tax rate  $\Upsilon_t$ ), raises lump-sum taxes, pays employment subsidy and issues nominal debt  $\mathcal{B}_t$ . The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{t+1} = (1 + i_t)(\mathcal{B}_t + P_t G_t - \Upsilon_t P_t Y_t - T + \mu^w).$$

This equation can be linearized as (defining  $B_t = \mathcal{B}_t/P_{t-1}$  and denoting the steady state ratio of debt to output as  $\chi$ ):

$$\chi \hat{B}_{t+1} = \chi \hat{i}_t + \frac{1}{\beta} \left( \chi \hat{B}_t - \chi \hat{\pi}_t + (1 - \theta) \hat{G}_t - \tau (\hat{\Upsilon}_t + \hat{Y}_t) \right). \quad (11)$$

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<sup>9</sup>The derivation is identical to the one in Woodford (2003), amended by the introduction of mark-up shocks as in Beetsma and Jensen (2004).

## 2.4 Aggregate Relationships

Output is distributed as wages and profits:

$$Y_t = W_t N_t + \Pi_t. \quad (12)$$

Government expenditures constitute part of demand, so the national income identity can be written as

$$Y_t = C_t + G_t, \quad (13)$$

and in steady state  $G = (1 - \theta)Y$ . The linearized national income identity is then:

$$\hat{Y}_t = (1 - \theta)\hat{G}_t + \theta\hat{C}_t. \quad (14)$$

## 2.5 Evolution of the Economy

We now write down the final system of equations for the ‘law of motion’ of the out-of-steady-state economy. We simplify notation by using lower case letters to denote ‘gap’ variables, where the gap is the difference between actual levels and natural levels i.e.  $x_t = \hat{X}_t - \hat{X}_t^n$ . The model consists of an intertemporal IS curve (15), the Phillips curve (16), national income identity (17), and an equation explaining the evolution of debt (18). It is:

$$c_t = \mathcal{E}_t c_{t+1} - \sigma(i_t - \mathcal{E}_t \pi_{t+1}), \quad (15)$$

$$\hat{\pi}_t = \beta \mathcal{E}_t \hat{\pi}_{t+1} + \frac{\kappa}{\sigma} \hat{C}_t + \frac{\kappa}{\psi} \hat{Y}_t + \frac{\kappa \tau}{(1 - \tau)} \hat{\tau}_t + \hat{\eta}_t, \quad (16)$$

$$y_t = (1 - \theta)g_t + \theta c_t, \quad (17)$$

$$\tilde{b}_{t+1} = \chi i_t + \frac{1}{\beta} \left( \tilde{b}_t - \chi \pi_t + (1 - \theta)g_t - \tau(\tau_t + y_t) \right), \quad (18)$$

where parameter  $\kappa = \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}$ . We denote  $\tilde{b}_t = \chi \hat{B}_t$ . It remains to specify policy.

## 2.6 Welfare Criterion

We assume that both authorities set their instruments to maximize the aggregate utility function:

$$\frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ u(C_s) + f(G_s) - \int_0^1 v(h_s(z)) dz \right]. \quad (19)$$

We show in the working paper version of this paper Blake and Kirsanova (2008) that problem (19) implies the following optimization problem for the policymakers. Each policymaker minimizes the discounted sum of all future losses:

$$L = \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} W_s^S, \quad (20)$$



where the one-period term is

$$W_s^S = \rho_c c_s^2 + \rho_g g_s^2 + \rho_y y_s^2 + \pi_s^2 + \mathcal{O}(\|\xi\|^3), \quad (21)$$

and  $\mathcal{O}(\|\xi\|^3)$  collects terms of higher order and we normalize the coefficient on inflation to one. Terms independent of policy, which are ignored, collect constant and linear and quadratic terms in shocks only. This quadratic approximation to social welfare is obtained assuming that there is a production subsidy  $\mu^w = T$  that eliminates the distortion caused by monopolistic competition and income taxes.<sup>10</sup>

Note that expression (21) contains a quadratic term in government spending,  $g$ . This term enters the welfare expression because it is assumed in (1) that households derive utility from the consumption of public goods, and that the steady state level of government spending reflects this. However, if we instead assumed that government spending was pure waste but the government still used  $g$  as a policy instrument, then changes in  $g$  would still influence social welfare through the national income identity, but it would not constitute an independent source of welfare loss.

Both policymakers minimize their own loss functions. If they are benevolent, each of them adopts the social loss function. Note that when assigning social welfare (21) to the monetary authority, we do not eliminate quadratic terms in government spending. This term is not independent of policy as monetary policy actions affect fiscal policy decisions. Each of the policymakers solves an optimization problem every period. The resulting optimal policy reactions lead to stochastic equilibria that should be compared across a suitable metric, which we take to be the microfounded social loss.

## 2.7 Optimal Discretionary Policy

We assume that both monetary and fiscal authorities act *non-cooperatively* in order to stabilize the economy against shocks. Both authorities are assumed to act under *discretion*. We assume that the monetary authority chooses the interest rate to minimize the welfare loss with per-period metrics *in the form of* (21) subject to system (15)-(18), and the always *benevolent* fiscal authority chooses spending to minimize the welfare loss (21) subject to the same system. Of course, if both authorities are benevolent then they both use the per-period social loss as their objective function. If the monetary authority is inflation-conservative then it can vary otherwise microfounded weight on inflation variability, so their per-period objective becomes:

$$W_s^M = \rho_c c_s^2 + \rho_g g_s^2 + \rho_y y_s^2 + \rho_\pi \pi_s^2 + \mathcal{O}(\|\xi\|^3), \quad (22)$$

where  $\rho_\pi \geq 1$ . Note that when assigning social welfare (22) to the monetary authority, we do not eliminate quadratic terms in government spending. This term is not independent of policy as monetary policy actions affect fiscal policy decisions.

We consider three leadership regimes: monetary or fiscal leadership and a Nash game. Our model belongs to the class of non-singular linear stochastic rational expectations model of the type described by Blanchard and Kahn (1980) augmented by a vector of control instruments.

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<sup>10</sup>This derivation follows Woodford (2003). The alternative way of deriving social welfare (Sutherland (2002) and Benigno and Woodford (2004)) is inappropriate for discretionary policy.

Specifically, the evolution of the economy is explained by the following system:

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_t^L \\ u_t^F \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}, \quad (23)$$

where  $y_t$  is a vector of predetermined variables with initial conditions  $y_0$  given,  $y_t = [\hat{\eta}_t, b_t]'$ ,  $x_t$  is a vector of non-predetermined (or jump) variables,  $x_t = [\pi_t, c_t]'$ .  $u_t^F$  and  $u_t^L$  are the two vectors of policy instruments of two policymakers, named  $F$  and  $L$ .  $u_t^F$  and  $u_t^L$  are either interest rate or spending (and tax) in our model.

The two policymakers have the following loss functions:

$$J_t^j = \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (g_s' \mathcal{Q}^j g_s), \quad (24)$$

where  $j = \{L, F\}$  and  $g_s$  is a vector of goal variables of the policymaker, which is a linear function of state variables and instruments,  $g_s = \mathcal{C} [y_s', x_s', u_s^{L'}, u_s^{F'}]'$ .

We demonstrate in Appendix A that in the linear quadratic framework optimal discretionary policy reaction function is necessarily presented in the form of linear rules with feedback coefficients on predetermined state variables. We show that a solution to a linear-quadratic optimization problem with two players subject to time consistency constraint can be written as a pair of linear rules

$$u_t^F = -F^F y_t - L u_t^L, \quad (25)$$

$$u_t^L = -F^L y_t, \quad (26)$$

where  $L = -\partial u_t^F / \partial u_t^L$ : in a leadership equilibrium the follower treats the leader's policy instrument parametrically and the leader can manipulate the follower. If the authorities play a Nash game then  $L \equiv 0$ .

Each of the policymakers solves an optimization problems every period. The resulting optimal policy reactions lead to stochastic equilibria that should be compared across a suitable metric. The obvious choice of this metric is the microfounded social loss.

Before we proceed we note that discretionary policy is known to generate multiple equilibria (Blake and Kirsanova (2007)) if there are endogenous predetermined state variables in the system. The presence of the debt accumulation process in our dynamic system could potentially lead to such a multiplicity. However, after an extensive search we did not find any equilibria other than those discussed in the paper and we conclude that multiplicity is not characteristic of either the model or the policy problem.

In what follows we assume that the fiscal authority uses government spending as an instrument. This choice is nearly arbitrary, but there are some considerations. First, there is no well established form for fiscal policy rule. Empirical estimates of fiscal policy reaction functions (see Favero and Monacelli (2005), Taylor (2000), Auerbach (2002) for example) suggest that both government spending and taxes are varied by the fiscal authority. Second, from a methodological point of view it is a convenient starting point. We discuss in Section 4 how our results change if we use taxes instead.

## 2.8 Calibration

We take the model's frequency to be quarterly. To achieve a steady state rate of interest of approximately 4%, we set the household discount rate  $\beta$  to 0.99. Output is normalized to one, and the ratio of government consumption to output,  $1 - \theta$ , is 0.25, which determines the relative preference for government spending in utility. The remaining parameters of the utility function are typical of those used in the literature, see e.g. Canzoneri et al. (2006). The elasticity of intertemporal substitution  $\sigma$  is taken as  $1/1.5$ , the Calvo parameter  $\gamma$  is set at 0.75 so as to imply average contracts of about a year, the elasticity of demand is taken as  $\varepsilon = 7.0$  to achieve a 17% mark up, and elasticity of labour demand is taken as  $\psi = 1/3$ .

We consider two values for the debt to GDP ratio. Our 'high' debt level,  $\chi$ , corresponds to 30% of annual output, which is still less than the level of debt in a number of European economies. However, we only consider one-period debt, so the figure of 30% is large enough to demonstrate *qualitative* difference with 0% debt, that we treat as 'low' debt level.

We only study the effect of cost-push shocks as they are the biggest potential source of social loss. We have examined what happens if we introduce taste or productivity shocks, and all quantitative results are virtually the same.

We calibrate the standard deviation of an i.i.d. cost-push shock as 0.005. In our base line case this generates a standard deviation for inflation of 0.0038, approximately the same order of magnitude as experienced in developed countries over the last couple of decades. This number is also not unreasonable given other academic studies. Ireland (2004) uses a cost-push shock, which is AR(1) with a standard deviation of 0.0044. Smets and Wouters (2003) reports an i.i.d. cost push shock with a much smaller standard deviation in the model with inflation persistence, while Rudebusch (2002) estimates a standard deviation of 0.01 for an i.i.d. cost push shock.

## 3 Two Policy Scenarios

In this section we demonstrate our main results by contrasting two policy scenarios. The first, benchmark case, is the case of benevolent policymakers. We look at how monetary and fiscal policy interact if policy objectives are not distorted. As we shall see, the qualitative results crucially depend on calibration of one particular parameter, the steady state level of debt. In what follows we find it convenient to emphasize the difference while we explain each of the policy scenarios that we consider. We, however, defer the remaining robustness analysis to Section 4. Having analyzed the benchmark case of benevolent policymakers we study the second scenario of interest: the conservative central bank scenario.

### 3.1 Benevolent Policymakers

The case of benevolent policymakers is a convenient benchmark and we start with it. We assume that both authorities have *identical* per-period *social* objectives (21). Figure 1 plots the impulse responses to a unit cost-push shock for the low and high debt cases.

The main results are the following:

1. All non-cooperative regimes with identical objectives deliver the same equilibrium.

This first (and probably unsurprising) result sets an important benchmark as it shows that strategic behavior can only be of importance if there is conflict in objectives. That the order of moves is of no importance is a direct consequence of policymakers having identical objectives and the uniqueness of the equilibrium. In this setting, neither authority is trying to exploit the other in a pursuit of their own target – they internalize externalities.<sup>11</sup>

2. The final policy mix very much depends on the *steady state level* of debt.

- a. In the low debt case contractionary monetary policy is supported by contractionary fiscal policy in a response to a positive cost push shock. (Both government spending and interest rate affect inflation via their effect on marginal cost.) This leads to a rise in debt. Fiscal policy is then used to bring debt back to the steady state: government spending stays at a reduced level for a long time.
- b. In the high debt case, the interest rate *falls* in the first period in a response to a positive cost push shock. As the first-order effect of interest rates on debt is large, a fall in interest rates reduces the level of domestic debt. Moreover, the fiscal contraction reduces debt even further. This first-period response allows monetary policy to raise interest rate in the second period and reduce inflation. (Note that the *second-period* inflation overshoots the steady state level. This helps to reduce inflation in the *first* period too, as the rational private sector sets prices lower in anticipation of this lower future price.<sup>12</sup>) Fiscal policy can also raise spending as this not only stabilizes debt, but also reduces recession caused by high interest rates.

The left panel in Figure 2 demonstrates that the social loss is a non-linear function of the steady state level of debt,  $\chi$ . This is consistent with a striking change in the way the stabilization policy works. When  $\chi$  is below some threshold level then any further rise creates more problems for an ‘active’ monetary policy combined with fiscal policy to both stabilize the economy and keep debt accumulation under control.<sup>13</sup> When  $\chi$  is above this threshold monetary policy becomes ‘passive’ and higher level of  $\chi$  requires a bigger rise in the interest rate in period two, which also curtails inflation. The two problems have the same solution hence the loss falls. To emphasize the difference we have chosen the low and high debt cases either side of the ‘hump’.<sup>14</sup>

3. The *dynamic* process of debt accumulation plays a very important role for the stabilization policy mix. First, its presence imposes the requirement on the policy mix to prevent an

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<sup>11</sup>Formally, it is straightforward to demonstrate in general case that Nash equilibrium with identical objectives coincides with cooperative equilibrium. The system of first order conditions for a single optimisation problem with two instruments in cooperative equilibrium will be identical to the two systems of FOCs for the two separate maximisation problems for each of the instruments. Similarly we can deal with Stackelberg equilibrium: the systems of FOCs will contain some additional terms, but they all be zero if objectives are the same.

<sup>12</sup>See Woodford (2003), pp. 486-501 for the discussion of a similar inflation overshooting under commitment policy.

<sup>13</sup>We use the term ‘active’ to identify monetary policy that increases interest rates in response to a cost-push shock and we use the term ‘passive’ otherwise. There is a resemblance of these two regimes to those identified in Leeper (1991), although we in our case fiscal policy has no tasks other than the stabilization of the economy.

<sup>14</sup>One might think that such a qualitative difference in monetary-fiscal interactions indicates the presence of multiple equilibria. We have checked that this is not the case.

explosion of debt. Second, its presence alters the dynamics of stabilization. If there is no debt, then following a shock a discretionary policy stabilizes the economy within *one* period. Policy can only reduce the *amplitude* of reactions to shocks within the first period. If debt accumulation is present then policy can also reduce the half-life of effects of shocks that have entered the system. In other words it enables stabilization to be smoothed over many periods, which may or may not be welfare improving. With the presence of debt the private sector's expectations affect the economy for more than one period, as the evolution of debt can be affected by the forecast of future policy. Expectations set in period  $t$  affect the whole future path of state variables and they affect policy decisions taken in period  $t + k$ ,  $k \geq 1$ . The way monetary policy stabilizes inflation in the case of high debt (point 2b above) is an example of how expectations of future policy can be exploited.

The social loss in the low and high debt cases is 2.5652 and 2.6007, correspondingly. We measure social loss as percentage of steady state consumption that should be given up in order to compensate for reduced volatility. All our future loss computations will be relative to these numbers.

This section has established that with benevolent policymakers strategic behavior is of no consequence. However, there are issues that arise from the dynamic nature of the model through the steady state level of debt. In what follows we need to investigate the importance of steady state levels of debt together with the conservative central bank proposal.

### 3.2 Conservative Central Bank

Suppose the fiscal authorities are benevolent, but we allow the monetary authorities to place a higher relative weight on inflation stabilization, by adopting objective (22) with  $\rho_\pi \geq 1$ . How does this affect social welfare? We keep the fiscal authorities benevolent.

The left hand side panel of Figure 3 suggests that the loss quickly rises with the degree of monetary conservatism for all three non-cooperative regimes. When the penalty is very close to one, and in the low debt case, there is an extremely small social gain for the fiscal leadership and the Nash regimes but the monetary leadership regime is unambiguously worse than the cooperation of benevolent authorities. If equilibrium debt is large then there is no social gain for any degree of inflation-conservatism and the social loss rises quickly with the degree of inflation conservatism. To understand these results it is instructive to look at impulse responses to a unit cost-push shock in Figure 4. We plot differences with the cooperative solution, which itself is plotted in Figure 1.

**Nash equilibrium.** In the *low debt* case the monetary authority reacts more actively to a cost push shock than if it were benevolent. It is more concerned with inflation variability than society and is prepared to pay for gains with higher variability of the demand-related components. The monetary authority, therefore, raises interest rate higher to eliminate inflation more aggressively. The fiscal authority is benevolent and tries to eliminate the resulting recession. It therefore contracts less (expands more) than if it were benevolent. Inflation is reduced, indeed. This reduction in inflation is only outweighs the cost of fiscal volatility if the degree of monetary

conservatism is extremely small. With higher degree of conservatism of the monetary authorities the implied fiscal volatility becomes very costly.

In the *high debt* case the monetary authority has to take into account the debt stabilization issues. It chooses to reduce interest rate in the first period by more, but the resulting smaller fiscal contraction allows the monetary authorities to raise interest rate by more in the second period, without having adverse effect on debt. Inflation, thus, overshoots more (falls relative to the cooperation case) in the second period. It, *therefore*, rises less (or falls relative to the cooperation case) in the first period. Again, the reason for its reduction in the first period is the second-period overshooting and rational expectations of price-setters: rational private sector perceives the second-period fall of prices and sets prices low in the first period.

The Nash game leads to more aggressive policy than under cooperation, and this results in lower inflation, but also in higher volatility of demand-related terms and instruments, and therefore, in more costly equilibrium.

**Fiscal leadership.** In the *low debt* case the fiscal authority knows that if it contracts less than in the cooperative scenario then this will cause the monetary authority to contract more in order to fight excess inflation. Moreover, the monetary authority will contract even more due to its inherent conservatism. So the fiscal authority chooses to contract only slightly less than in the cooperative scenario. The monetary authority still contracts *more* than in the cooperative scenario and overall this results in slightly lower inflation. Inflation falls nearly as much as under Nash, because fiscal authorities do not expand as much as under Nash. Debt rises only slightly higher than in the cooperative scenario, and less than if both authorities play Nash. With higher degree of monetary conservatism, monetary authorities have to pay too high output and consumption cost for smaller inflation, so the regime is unambiguously worse than the cooperation.

In the *high debt* case the authorities have more problems with debt stabilization. The fiscal authority contracts more in the first period thus not letting monetary policy to expand as much as under Nash. The monetary authority *therefore* contracts less in the second period. This creates less inflation overshooting than under Nash. The loss is bigger than under Nash for moderate degree of monetary conservatism. However, with higher degree of conservatism the joint monetary-fiscal interactions result in smaller consumption volatility than under Nash and this improves welfare slightly. The regime becomes slightly better than Nash, although the quantitative response is extremely close to the Nash outcome. The debt stabilization task nearly equally constrains instrument movements in all three non-cooperative regimes.

**Monetary leadership.** In the *low debt* case the leading monetary authority knows that raising the interest rate causes the fiscal authority to try to offset the resulting recession. This consideration would stop the *benevolent* monetary authority from raising interest rates. However, as the monetary authority is conservative, the cost of moving  $g$  becomes *relatively* less important, so it does raise the interest rate. But it raises the interest rate less than in a Nash game. Lower pressure on debt allows fiscal policy to offset the effect more efficiently. Inflation falls less than under Nash and this determines the loss.

In the *high debt* case the monetary authority contracts more in the first period than under Nash and the fiscal authorities have to reduce spending more too. The debt does not fall as

much as it does under Nash. The monetary authority is unable to achieve as large inflation overshooting in the second period, and thus overall gain in inflation stabilization, as under Nash. It is able, however, to achieve smaller variability of demand-related terms than under Nash. This results in smaller welfare loss than under Nash.

## 4 Robustness: Using Tax as an Instrument

### 4.1 Tax as fiscal instrument

We have shown how our results change with the level of steady state debt. The other important choice is that of the instrument for fiscal policy. We rerun our simulations assuming that (i) fiscal policy uses tax as the instrument and (ii) fiscal policy uses two instruments, both income tax rate and spending. Figure 5, which repeats Figure 3 but is plotted for taxes (the left hand side panels) and taxes and spending (the right hand side panels), suggests that our main conclusion remains valid: generally speaking, delegating monetary policy to a conservative central bank does not improve social welfare.<sup>15</sup>

However, there are some important differences with the case where only spending is used. Consider using tax as a single fiscal instrument and suppose that authorities are *benevolent*. As taxes affect marginal cost directly, fiscal policy tries to offset any cost-push shocks immediately by lowering tax rate. The first column of plots in Figure 6 suggest that in response to a unit cost push shock taxes fall that allows monetary policy to raise interest rate without any solvency concerns.<sup>16</sup> The debt is also stabilized *by taxes*: the first-period reduction in tax rate and high interest rate require higher taxes in consequent periods; taxes rise and bring debt back to the steady state nearly very quickly. The dynamics of debt is very different from the one in the case where fiscal policy can only use spending as an instrument: debt is returned back to the steady state within several periods. Taxes have no direct effect on demand and consumption, so fiscal policy can more efficiently eliminate debt displacement in consequent periods, with less externalities, and monetary policy can efficiently stabilize inflation. However, the welfare gain from using taxes instead of spending is very small: if the loss under fully optimal monetary-fiscal policy is 2.5652 when only spending are used, then it is only marginally lower (2.5596) if taxes is the only instrument.<sup>17</sup> This is because we look at policy *under discretion*. Benigno and Woodford (2004) demonstrate that under commitment fiscal policy is very successful and nearly offsets their effects on the system. Under discretion, however, taxes cannot move as freely as under commitment, and they cannot efficiently offset cost-push shocks, although they still move in the right direction. The right panel in Figure 2 suggests that with higher level of steady state debt,  $\chi$ , the social loss rises. The level of  $\chi$  determines the size of the first-order effect of interest rate on debt accumulation. With higher  $\chi$  the problem of debt stabilization becomes more difficult. Taxes become predominantly used in the debt stabilization task and make the task of inflation stabilization more difficult for monetary policy.

The relative ranking of different leadership regimes barely changes, however. Nash regime

<sup>15</sup>Here and below we say that fiscal policy uses taxes when income tax rate changes.

<sup>16</sup>We have checked that impulse responses are qualitatively similar for all degrees of conservatism and for low and high debt scenarios. So we only present small conservatism low debt case in Figure 6.

<sup>17</sup>In all cases we assume that the initial debt is at its steady state level.

leads to most welfare losses. However, it is not the most aggressive regime this time. The second column of plots in Figure 6 illustrate this. As before, we plot impulse responses relative to those under cooperation of benevolent policymakers. As all qualitative responses are very similar for different degree of conservatism and for the low and high debt scenarios we only plot the case with low conservatism and with low level of debt.

The second column of plots in Figure 6 demonstrates interactions for different leadership regimes in the case of *conservative central bank*. Under the *Nash regime*, monetary authorities raise interest rate higher than if they were benevolent. They reduce inflation by more but they also reduce consumption. The optimal response of fiscal authority is to raise taxes. If monetary policy is not taken into account, then higher taxes allow the cost-push shock to have bigger effect on inflation, real interest rate falls and consumption rises. In the Nash game the fiscal authority, therefore, reduce taxes by less than they do under cooperation. Monetary policy rises interest rate higher and so on. In equilibrium inflation is slightly reduced but consumption falls. For small degree of conservatism the gain of lower inflation outweighs the loss of volatile consumption and output, but consumption volatility rises very fast with the degree of monetary conservatism and the regime quickly becomes welfare inferior.

If the *monetary authority leads*, it knows that the fiscal authority will not reduce taxation as if it were benevolent, so it raises interest rate by less and the fiscal authority reduce tax by less than under Nash. This leads to nearly the same outcome as under Nash if the degree of conservatism is small. With higher degree of conservatism policy aggressiveness leads to volatility of consumption, which is still less than under Nash. So the monetary leadership regime is preferable over the Nash regime.

If the *fiscal authority leads*, it knows that the monetary authority will raise interest rates so it reduces taxes by more. It explicitly exploits the monetary policy reaction function: the monetary authority does raise interest rate, but by less if it were benevolent as the fiscal authority does part of disinflation. As a result of these first period movements debt rises high. Fiscal policy thus has to raise taxes in the second period, that also results in higher inflation in the second period. If the degree of monetary conservatism is small then the fiscal leadership regime delivers lowest loss among the three non-cooperative regime, because of the relatively large fall in inflation. The second period inflation hike, however, determines the large social welfare loss if monetary authorities have larger degree of conservatism.

Neither of non-cooperative regimes can be defined as most aggressive here: the Nash regime results in higher variability in interest rate while the fiscal leadership regime results in the biggest volatility of fiscal instrument.

## 4.2 Tax and spending as fiscal instruments

Adding government spending does not change any of the quantitative results as the last two columns of plots in Figure 3 demonstrate. An inflation-conservative monetary authority, generally speaking, generates social welfare loss. Adding government spending as a second instrument for fiscal policy does not change much, see the right hand side panels of Figure 6. If the authorities are benevolent then the social loss is 2.5441 in the low debt case, which is only marginally smaller than the loss from stabilization where taxes only are used. As debt is now stabilized by taxes, spending are optimally chosen to help monetary policy to reduce inflation if the authorities are



benevolent. If there are distorted objectives, then interest rate-tax interactions are nearly the same as if tax were the only fiscal instrument. The possibility to use spending as well does little difference: spending plays the same role (relative to the case of cooperation of benevolent policymakers) as in the scenario where it was the only fiscal instrument, one can compare the last column in Figure 6 with the first column of plots in Figure 4.

## 5 Summary of Results and Conclusions

This paper evaluates a conservative-central-bank proposal in a dynamic model with separate non-cooperative monetary and fiscal authorities which act under discretion. We demonstrate that if the fiscal policymaker is benevolent but acts strategically, then delegating monetary policy to a monetary policymaker that puts a higher than socially optimal weight on inflation stabilization usually increases stabilization bias and reduces social welfare. A distortion from socially optimal objectives of one of the authorities by changing the relative weights nearly always leads to social welfare losses in our model. Such distortion makes the two policymakers ‘fight’ each other that reduces social welfare nearly always for our model. What works well in an economy with one strategic policymaker may not work at all in an economy with two strategic policymakers, as the second strategic policymaker offsets all actions of the first one and vice versa.

Of course, this result does not imply that the problem of optimal delegation has no solution if there are several strategic policymakers. In our policy scenario we have distorted the relative weights on social objectives, but we did not introduce any additional objectives. Additional terms in policymakers’ objectives, like instrument smoothing costs, might have a different effect on the ability and willingness of the policymakers to engage in conflict with each other. Our result, however, suggests that making fiscal authorities flexible and strategic may have costs, and they should be taken into account.

To arrive to our main conclusions, we have investigated monetary and fiscal policy interactions under discretion in a fairly standard model with an explicit budget constraint for the fiscal authority. We have developed the necessary tools to analyze non-cooperative games between two policymakers and a rational private sector. There are also some additional conclusions that can be drawn from this analysis.

1. The level of steady state debt to output ratio has an important qualitative effect on policy interactions. In the high debt case policy has to take into account its effects on debt stabilization. This is particularly important for the monetary authority as the first-order effect of interest rates on debt accumulation rises substantially.
2. The choice of fiscal instrument is important, although not for assessing gains from delegation where there are losses whatever the fiscal instrument. The transmission mechanism differs considerably. Taxes are most useful in stabilizing debt. Spending has more limited powers to stabilize debt – and therefore monetary policy must be more associated with debt – but has a large effect on domestic demand.
3. Among the three non-cooperative regimes, the Nash equilibrium is unlikely to be welfare-dominating. In most case this regime leads to large movements of policy instruments that

typically implies large volatility of key economic variables. Most of our results suggest that monetary leadership is relatively better if the monetary authority is inflation-conservative, as in this model the inflation stabilization target has far more bigger weight in social objectives than any other targets.

Finally, we this paper offers an additional contribution to the literature: we offer a methodological approach to solving a non-cooperative leadership equilibria in which monetary and fiscal authorities may have different objectives. This approach can be easily modified to study different types of equilibria and games with many players in a multi-country setting.

## A Leadership Equilibrium Under Discretion

This section demonstrates how to solve non-cooperative dynamic games in linear-quadratic framework. The next section A.1 describes the general class of models we deal with. Currie and Levine (1993) demonstrate how to solve Nash games. We therefore only describe leadership equilibria.

### A.1 A Class of Models

We assume a non-singular linear stochastic rational expectations model of the type described by Blanchard and Kahn (1980) augmented by a vector of control instruments. Specifically, the evolution of the economy is explained by the following system:

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_t^L \\ u_t^F \end{bmatrix}, \quad (27)$$

where  $y_t$  is an  $n_1$ -vector of predetermined variables with initial conditions  $y_0$  given,  $x_t$  is  $n_2$ -vector of non-predetermined (or jump) variables,  $u_t^F$  and  $u_t^L$  are the two vectors of policy instruments of two policymakers, named  $F$  and  $L$ , of size  $k_F$  and  $k_L$  respectively. For notational convenience we define the  $n$ -vector  $z_t = (y_t', x_t')'$  where  $n = n_1 + n_2$ , and the  $k$ -vector of control variables  $u_t = (u_t^L', u_t^F')'$ , where  $k = k_F + k_L$ .

Typically, this system represents the solved out optimization problem for the ultimate follower in the policy game. This player also has ‘instruments’, represented by  $x_t$ , so the second equation in (27) is essentially its reaction function. Additionally, there is an equation explaining evolution of predetermined variable  $y_t$ .

The two policymakers have the following loss functions:

$$J_t^i = \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (g_s' \mathcal{Q}^i g_s) = \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (z_s' Q^i z_s + 2z_s' P^i u_s + u_s' R^i u_s), \quad (28)$$

where  $i = \{L, F\}$  and  $g_s$  is a vector of target variables of the policymaker,  $g_s = \mathcal{C}(z_s', u_s')'$ . The loss function of each player can include instrument costs of both players, but no assumptions about the invertibility of  $R^i$  are made.

It should be noted that none of the results depend on the deterministic setup outlined and the consequent assumption of perfect foresight. We can add an appropriate vector of shocks, but this unnecessarily complicates the analysis.<sup>18</sup>

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<sup>18</sup>Shocks can be included into vector  $y_t$ , see e.g. Anderson et al. (1996).

Before we proceed, we claim (and show this later) that a solution to a linear-quadratic optimization problem with two players subject to time consistency constraint can be written as a pair of linear rules

$$u_t^F = -F^F y_t - L u_t^L, \quad (29)$$

$$u_t^L = -F^L y_t, \quad (30)$$

where  $L = -\partial u_t^F / \partial u_t^L$  : in a leadership equilibrium the follower treats the leader's policy instrument parametrically.

Substitute (29) and (30) into (27) and obtain the evolution of the system under control:

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} - B_{12}(F^F - LF^L) - B_{11}F^L & A_{12} \\ A_{21} - B_{22}(F^F - LF^L) - B_{21}F^L & A_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix}. \quad (31)$$

## A.2 Reaction Function of the Private Sector

Suppose the reaction of the private sector is a linear function of the predetermined state variables:

$$x_t = -N y_t. \quad (32)$$

We can bring this representation into an equivalent form in terms of predetermined variables and controls (as did Oudiz and Sachs, 1985). Leading (32) one period we can substitute for  $y_{t+1}$  from the first equation in (27) to obtain:

$$x_{t+1} = -N y_{t+1} = -N(A_{11} y_t + A_{12} x_t + B_1 u_t^F + B_{11} u_t^L). \quad (33)$$

Combining this with the second equation in (27) we can easily obtain:

$$\begin{aligned} x_t &= -(A_{22} + N A_{12})^{-1} [(A_{21} + N A_{11}) y_t + (B_{22} + N B_1) u_t^F + (B_{21} + N B_{11}) u_t^L] \\ &= -J y_t - K^F u_t^F - K^L u_t^L, \end{aligned} \quad (34)$$

where

$$J = (A_{22} + N A_{12})^{-1} (A_{21} + N A_{11}), \quad (35)$$

$$K^F = (A_{22} + N A_{12})^{-1} (B_{22} + N B_1), \quad (36)$$

$$K^L = (A_{22} + N A_{12})^{-1} (B_{21} + N B_{11}). \quad (37)$$

Formula (34) describes the reaction function of the private sector as a linear reaction rule, with (so far) unknown coefficients  $J$ ,  $K^F$  and  $K^L$ . This representation is convenient for the rest of what follows. Using these definitions of  $J$  and  $K^F$ ,  $K^L$  we can write

$$y_{t+1} = (A_{11} - A_{12} J) y_t + (B_{11} - A_{12} K^L) u_t^L + (B_{12} - A_{12} K^F) u_t^F = A_J y_t + B_K^L u_t^L + B_K^F u_t^F, \quad (38)$$

where the definitions of  $J$ ,  $K$ ,  $A_J$  and  $B_K^L$ ,  $B_K^F$  will be convenient for the rest of what follows.

Before we move further note that, by construction,  $N$  satisfies the following continuous equation:

$$\begin{aligned} N &= J - K^F F^F + K^F L F^L - K^L F^L = \\ &= (A_{22} + N A_{12})^{-1} (A_{21} + N A_{11} - (B_{22} + N B_1)(F^F - L F^L) - (B_{21} + N B_{11}) F^L). \end{aligned} \quad (39)$$

### A.3 The Follower's Optimization Problem

The follower's cost-to-go from time  $t$  satisfies Bellman equation:

$$W_t = \frac{1}{2}(Z_t' Q^F Z_t + 2Z_t' P^F U_t + U_t' R^F U_t) + \beta \mathcal{E}_t W_{t+1}. \quad (40)$$

There welfare loss from time  $t$  must be a quadratic function of the predetermined state variables:

$$W_t = \frac{1}{2}(y_t' S y_t). \quad (41)$$

We can substitute this into (40) and using (34) and (38) we obtain:

$$\begin{aligned} y_t' S y_t = & y_t'(Q^* + \beta A_J' S A_J) y_t + u_t^{F'}(U_F^{*'} + \beta B_K^{F'} S A_J) y_t + y_t'(U_F^* + \beta A_J' S B_K^F) u_t^F \\ & + u_t^{L'}(U_L^{*'} + \beta B_K^{L'} S A_J) y_t + y_t'(U_L^* + \beta A_J' S B_K^L) u_t^L + u_t^{F'}(\beta B_K^{F'} S B_K^F + R^*) u_t^F \\ & + u_t^{F'}(\beta B_K^{F'} S B_K^L + P^{*'}) u_t^L + u_t^{L'}(\beta B_K^{L'} S B_K^F + P^*) u_t^F + u_t^{L'}(\beta B_K^{L'} S B_K^L + T^*) u_t^L, \end{aligned} \quad (42)$$

where  $Q^*, U_F^*, U_L^*, R^*, P^*$  and  $T^*$  are defined by

$$Q^* = Q_{11}^F - Q_{12}^F J - J' (Q_{21}^F - Q_{22}^F J), \quad (43)$$

$$U_F^* = (J' Q_{22}^F - Q_{12}^F) K^F + P_{12}^F - J' P_{22}^F, \quad (44)$$

$$U_L^* = (J' Q_{22}^F - Q_{12}^F) K^L + P_{11}^F - J' P_{21}^F, \quad (45)$$

$$R^* = K^{F'} (Q_{22}^F K^F - P_{22}^F) + R_{22}^F - P_{22}^{F'} K^F, \quad (46)$$

$$P^* = K^{L'} (Q_{22}^F K^F - P_{22}^F) + R_{12}^F - P_{21}^{F'} K^F, \quad (47)$$

$$T^* = K^{L'} (Q_{22}^F K^L - P_{21}^F) - P_{21}^{F'} K^L + R_{11}^F, \quad (48)$$

and matrices  $Q, P$  and  $R$  are partitioned conformably with  $z_s = (y_s', x_s')'$  and  $u_s = (u_s^L, u_s^F)'$ .

The feedback rule can be determined from (42) by differentiating the loss function with respect to  $u_t^F$ :

$$u_t^F = -(R^* + \beta B_K^{F'} S B_K^F)^{-1} ((U_F^{*'} + \beta B_K^{F'} S A_J) y_t + (P^{*'} + \beta B_K^{F'} S B_K^L) u_t^L).$$

It follows that  $F^F$  and  $L$  are defined by

$$F^F = (R^* + \beta B_K^{F'} S B_K^F)^{-1} (U_F^{*'} + \beta B_K^{F'} S A_J), \quad (49)$$

$$L = (R^* + \beta B_K^{F'} S B_K^F)^{-1} (P^{*'} + \beta B_K^{F'} S B_K^L). \quad (50)$$

Now, we substitute the reaction rules (29), (30), (49) and (50) in (42) and obtain that  $S$  satisfies

$$\begin{aligned} S = & \beta (A_J - B_K^L F^L)' S (A_J - B_K^L F^L) \\ & - ((U_F^{*'} - P^{*'} F^L) + \beta B_K^{F'} S (A_J - B_K^L F^L))' (R^* + \beta B_K^{F'} S B_K^F)^{-1} \\ & \times ((U_F^{*'} - P^{*'} F^L) + \beta B_K^{F'} S (A_J - B_K^L F^L)) + (Q^* - F^{L'} U_L^{*'} - U_L^* F^L + F^{L'} T^* F^L). \end{aligned} \quad (51)$$

If we find  $S$  then  $F^F$  and  $L$  are uniquely determined.

#### A.4 The Leader's Optimization Problem

The leader takes into account the follower's reaction function. Substitute rule (29) into equation (27) and obtain that the system under control evolves as:

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix} [u_t^L], \quad (52)$$

where  $\tilde{A}, \tilde{B}$  are defined as:

$$\tilde{J} = J - K^F F^F, \quad \tilde{K} = K^L - K^F L, \quad (53)$$

$$\tilde{A}_{11} = A_{11} - B_{12} F^F, \quad \tilde{A}_{12} = A_{12}, \quad \tilde{A}_{21} = A_{21} - B_{22} F^F, \quad \tilde{A}_{22} = A_{22}, \quad (54)$$

$$\tilde{B}_1 = B_{11} - B_{12} L, \quad \tilde{B}_2 = B_{21} - B_{22} L, \quad \tilde{A}_J = \tilde{A}_{11} - \tilde{A}_{12} \tilde{J}, \quad \tilde{B}_K = \tilde{B}_1 - \tilde{A}_{12} \tilde{K}. \quad (55)$$

Substitute rule (29) into the leader's loss function to obtain the new penalty matrix determined by

$$\tilde{Q}_{11} = Q_{11}^L - P_{12}^L F^F - F^{F'} (P_{12}^{L'} - R_{22}^L F^F), \quad (56)$$

$$\tilde{Q}_{12} = \tilde{Q}_{21} = Q_{12}^L - F^{F'} P_{22}^{L'}, \quad \tilde{Q}_{22} = Q_{22}^L, \quad (57)$$

$$\tilde{P}_1 = P_{11}^L - P_{12}^L L - F^{F'} (R_{21}^L - R_{22}^L L), \quad (58)$$

$$\tilde{P}_2 = P_{21}^L - P_{22}^L L, \quad \tilde{R} = R_{11}^L - R_{12}^L L - L' (R_{21}^L - R_{22}^L L). \quad (59)$$

Substitute rule (29) into reaction (34) to obtain

$$x_t = -\tilde{J}y_t - \tilde{K}u_t^L, \quad (60)$$

where  $\tilde{J}$  and  $\tilde{K}$  are defined in (53).

Similar to derivations in Section A.3, the cost-to-go from time  $t$  satisfies the following dynamic programming equation:

$$W_t = \frac{1}{2} (z_t' \tilde{Q} z_t + 2z_t' \tilde{P} u_t^L + u_t^{L'} \tilde{R} u_t^L) + \beta \mathcal{E}_t W_{t+1}. \quad (61)$$

The welfare loss at time  $t$  is given by  $W_t = \frac{1}{2} (y_t' \tilde{S} y_t)$ . We substitute it into formula (61) and, using (60) and  $y_{t+1} = \tilde{A}_J y_t + \tilde{B}_K u_t^L$ , obtain:

$$\begin{aligned} y_t' \tilde{S} y_t &= y_t' (\tilde{Q}^* + \beta \tilde{A}_J' \tilde{S} \tilde{A}_J) y_t + u_t^{L'} (\tilde{U}^{*'} + \beta \tilde{B}_K' \tilde{S} \tilde{A}_J) y_t \\ &\quad + y_t' (\tilde{U}^* + \beta \tilde{A}_J' \tilde{S} \tilde{B}_K) u_t^L + u_t^{L'} (\beta \tilde{B}_K' \tilde{S} \tilde{B}_K + \tilde{R}^*) u_t^L, \end{aligned} \quad (62)$$

where  $\tilde{A}_J, \tilde{B}_K, \tilde{Q}^*, \tilde{U}^*, \tilde{R}^*$  are determined by (55) and

$$\tilde{Q}^* = \tilde{Q}_{11} - \tilde{Q}_{12} \tilde{J} - \tilde{J}' \tilde{Q}_{21} + \tilde{J}' \tilde{Q}_{22} \tilde{J}, \quad (63)$$

$$\tilde{U}^* = \tilde{J}' \tilde{Q}_{22} \tilde{K} - \tilde{Q}_{12} \tilde{K} + \tilde{P}_1 - \tilde{J}' \tilde{P}_2, \quad (64)$$

$$\tilde{R}^* = \tilde{K}' \tilde{Q}_{22} \tilde{K} + \tilde{R} - \tilde{K}' \tilde{P}_2 - \tilde{P}_2' \tilde{K}. \quad (65)$$

The feedback rule can be determined from (62) by differentiating the loss function with respect to  $u_t^L$ :

$$u_t^L = -(\tilde{R}^* + \beta \tilde{B}'_K \tilde{S} \tilde{B}_K)^{-1} (\tilde{U}^{*'} + \beta \tilde{B}'_K \tilde{S} \tilde{A}_J) y_t.$$

From where  $F^L$  is determined by

$$F^L = \left( \tilde{R}^* + \beta \tilde{B}'_K \tilde{S} \tilde{B}_K \right)^{-1} \left( \tilde{U}^{*'} + \beta \tilde{B}'_K \tilde{S} \tilde{A}_J \right). \quad (66)$$

Substitute (30) and (66) into formula (62) we obtain that  $\tilde{S}$  satisfies

$$\tilde{S} = \tilde{Q}^* + \beta \tilde{A}'_J \tilde{S} \tilde{A}_J - \left( \beta \tilde{B}'_K \tilde{S} \tilde{A}_J + \tilde{U}^{*'} \right)' \left( \tilde{R}^* + \beta \tilde{B}'_K \tilde{S} \tilde{B}_K \right)^{-1} \left( \beta \tilde{B}'_K \tilde{S} \tilde{A}_J + \tilde{U}^{*'} \right). \quad (67)$$

Again, if we know  $\tilde{S}$  then  $F^L$  is uniquely determined.

## A.5 Leadership equilibrium

We define discretionary leadership equilibrium as a set of five matrices  $\{F^L, F^F, L, N, S, \tilde{S}\}$  such that the triplet  $\{N, S, \tilde{S}\}$  is as an asymptotically stable steady state of the following difference system

$$N_{s+1} = (A_{22} + N_s A_{12})^{-1} (A_{21} + N_s A_{11} - (B_{22} + N_s B_{12}) (F_s^F - L_s F_s^L) - (B_{21} + N_s B_{11}) F_s^L), \quad (68)$$

$$\begin{aligned} S_{s+1} = & \beta (A_{Js} - B_{Ks}^L F_s^L)' S_s (A_{Js} - B_{Ks}^L F_s^L) - (U_{Fs}^{*'} - P_s^{*'} F_s^L + \beta B_{Ks}^{F'} S_s (A_{Js} - B_{Ks}^L F_s^L))' \\ & \times (R_s^* + \beta B_{Ks}^{F'} S_s B_{Ks}^F)^{-1} (U_{Fs}^{*'} - P_s^{*'} F_s^L + \beta B_{Ks}^{F'} S_s (A_{Js} - B_{Ks}^L F_s^L)) \\ & + Q_s^* - F_s^{L'} U_{Ls}^{*'} - U_{Ls}^* F_s^L + F_s^{L'} T_s^* F_s^L, \end{aligned} \quad (69)$$

$$\begin{aligned} \tilde{S}_{s+1} = & \tilde{Q}_s^* + \beta \tilde{A}'_{Js} \tilde{S}_s \tilde{A}_{Js} - \left( \beta \tilde{B}'_{Ks} \tilde{S}_s \tilde{A}_{Js} + \tilde{U}_s^{*'} \right)' \\ & \times \left( \tilde{R}_s^* + \beta \tilde{B}'_{Ks} \tilde{S}_s \tilde{B}_{Ks} \right)^{-1} \left( \beta \tilde{B}'_{Ks} \tilde{S}_s \tilde{A}_{Js} + \tilde{U}_s^{*'} \right), \end{aligned} \quad (70)$$

where all coefficients with subscript  $s$  are (non-) linear functions of  $S_s, \tilde{S}_s$  and  $N_s$  and defined above *ref*. If we find an asymptotically stable steady state  $\{N, S, \tilde{S}\}$  then corresponding triplet  $\{F^L, F^F, L\}$  defines policy reactions in this equilibrium.

System (68)-(70) is a non-linear system of difference matrix equations. It may have many solutions. We demonstrate in Blake and Kirsanova (2007) that even simpler system of this kind (that defines discretionary equilibrium with only one policymaker) does have multiple equilibria in certain cases. In this paper we admit such possibility and do everything to locate possible multiple solutions numerically.

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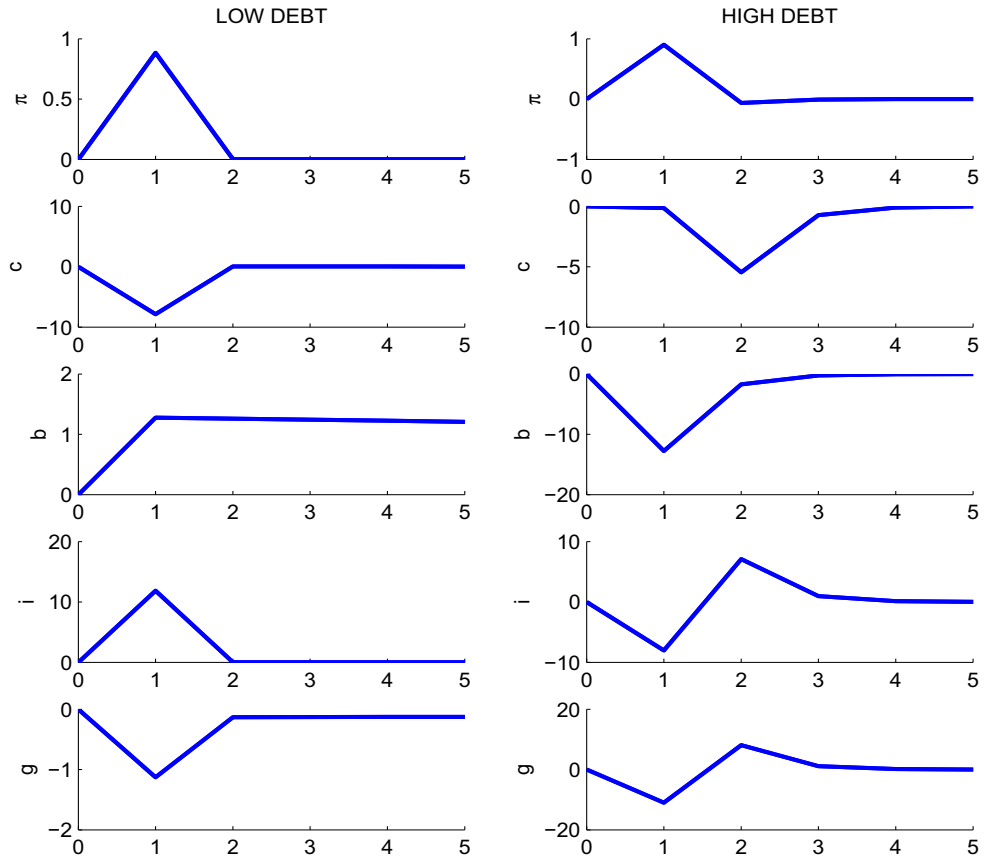


Figure 1: Impulse responses to a unit cost-push shock under full optimization of benevolent authorities. Fiscal policy uses spending as an instrument.

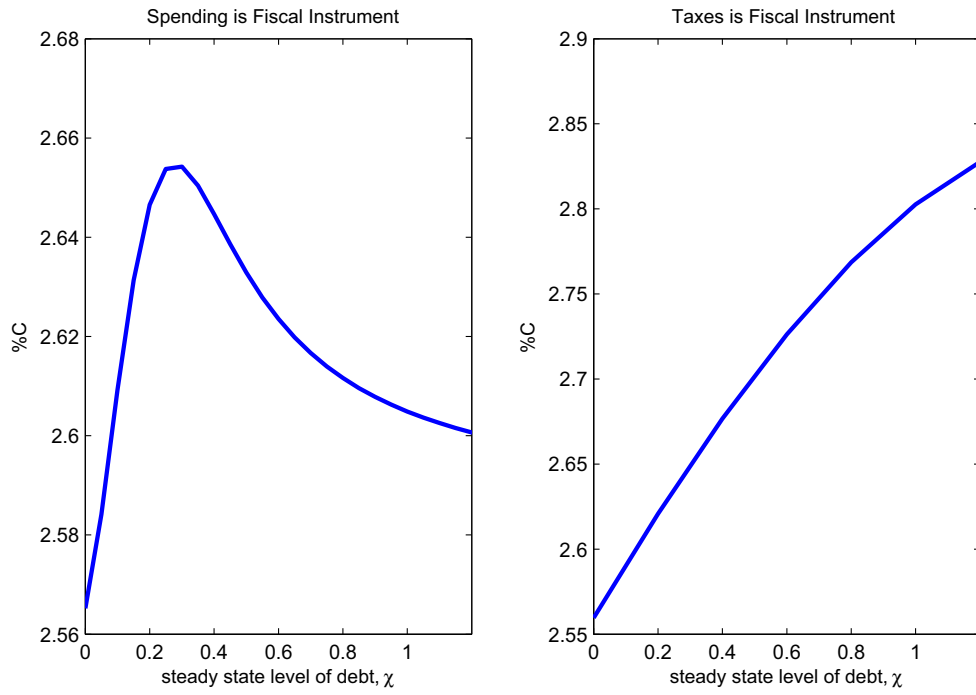


Figure 2: Joint monetary-fiscal optimisation, benevolent policymakers. The social loss as a function of steady state level of debt. Different fiscal instruments.

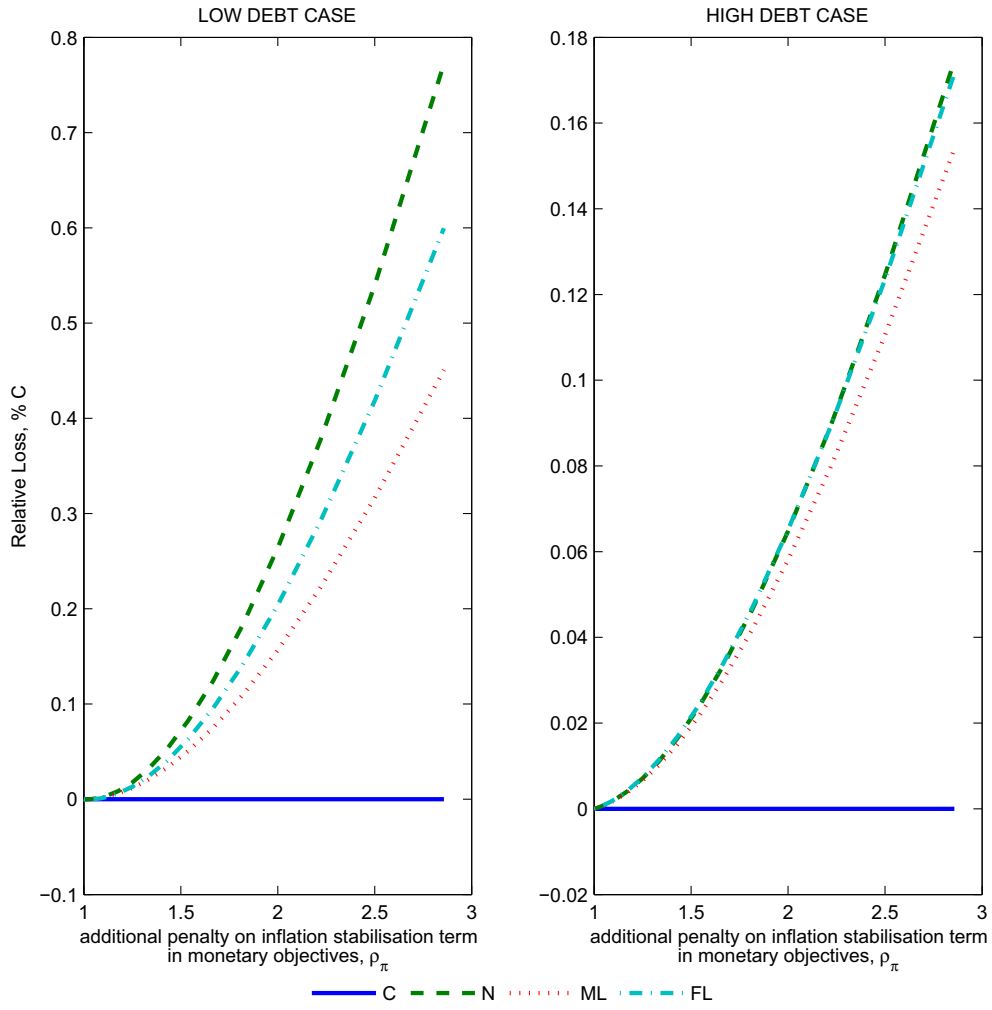


Figure 3: Social welfare loss as a function of monetary conservatism for different non-cooperative regimes. Fiscal policy uses spending as an instrument.

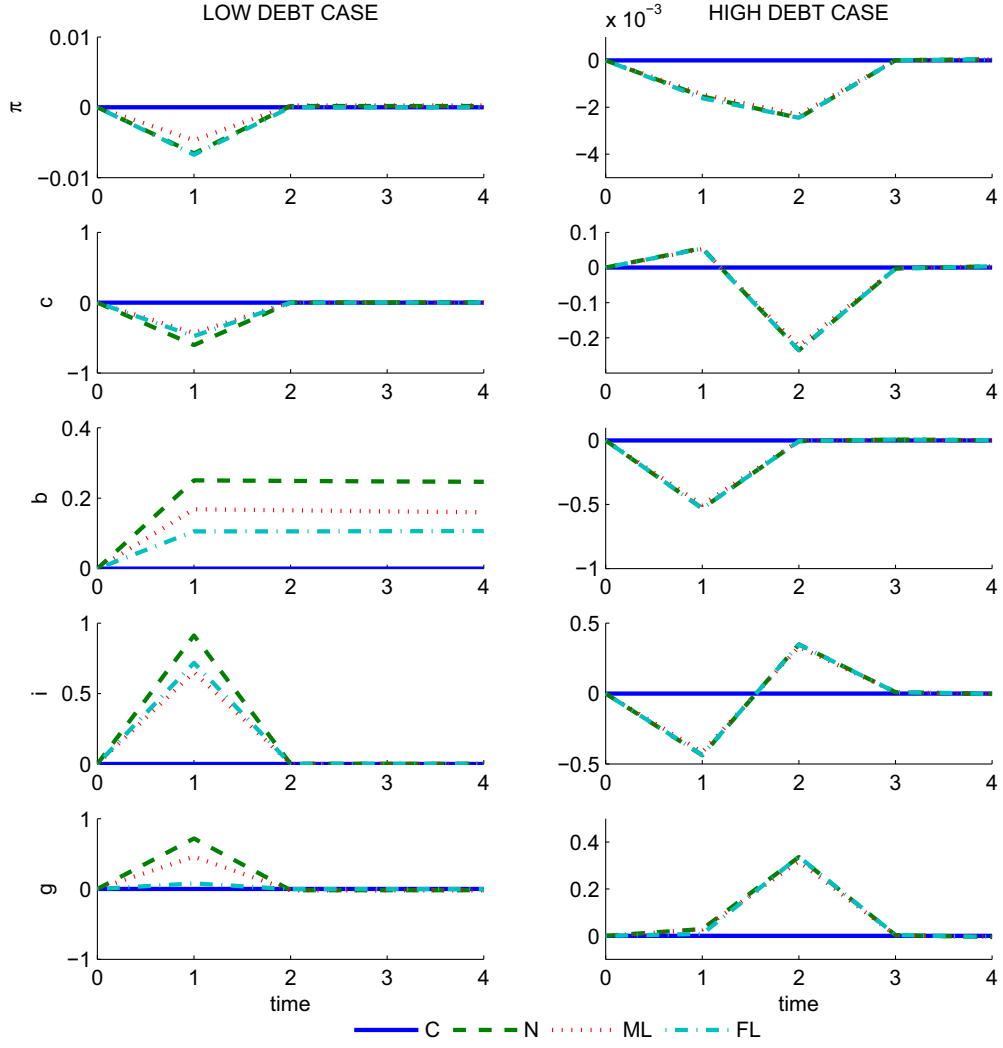


Figure 4: Impulse responses to a unit cost-push shock for different strategic regimes if the monetary authority is conservative with small degree of conservatism,  $\rho_\pi = 1.06$ . Fiscal policy uses spending as an instrument. All responses shown as relative to those under cooperation.

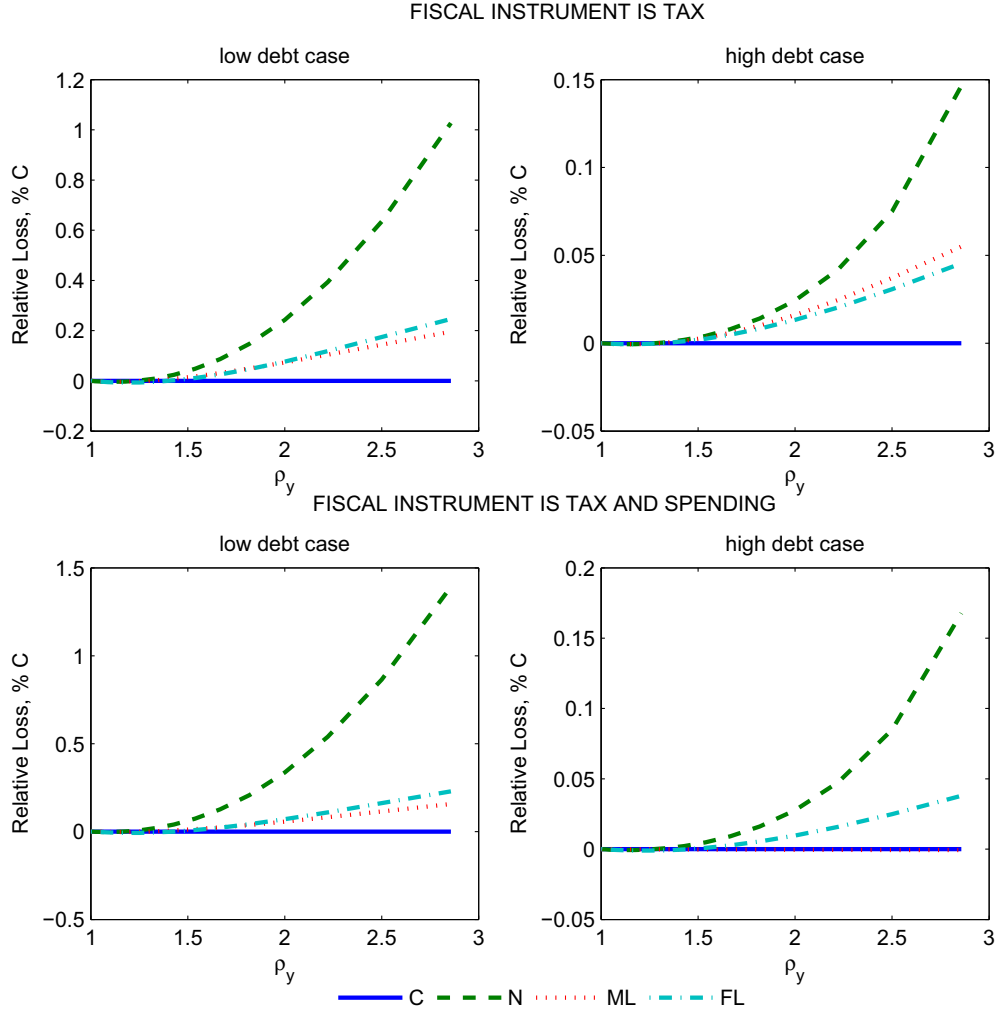


Figure 5: Social loss as a function of conservatism for different strategic regimes and policy arrangements: fiscal policy uses taxes as an instrument or it uses both instruments, taxes and spending.

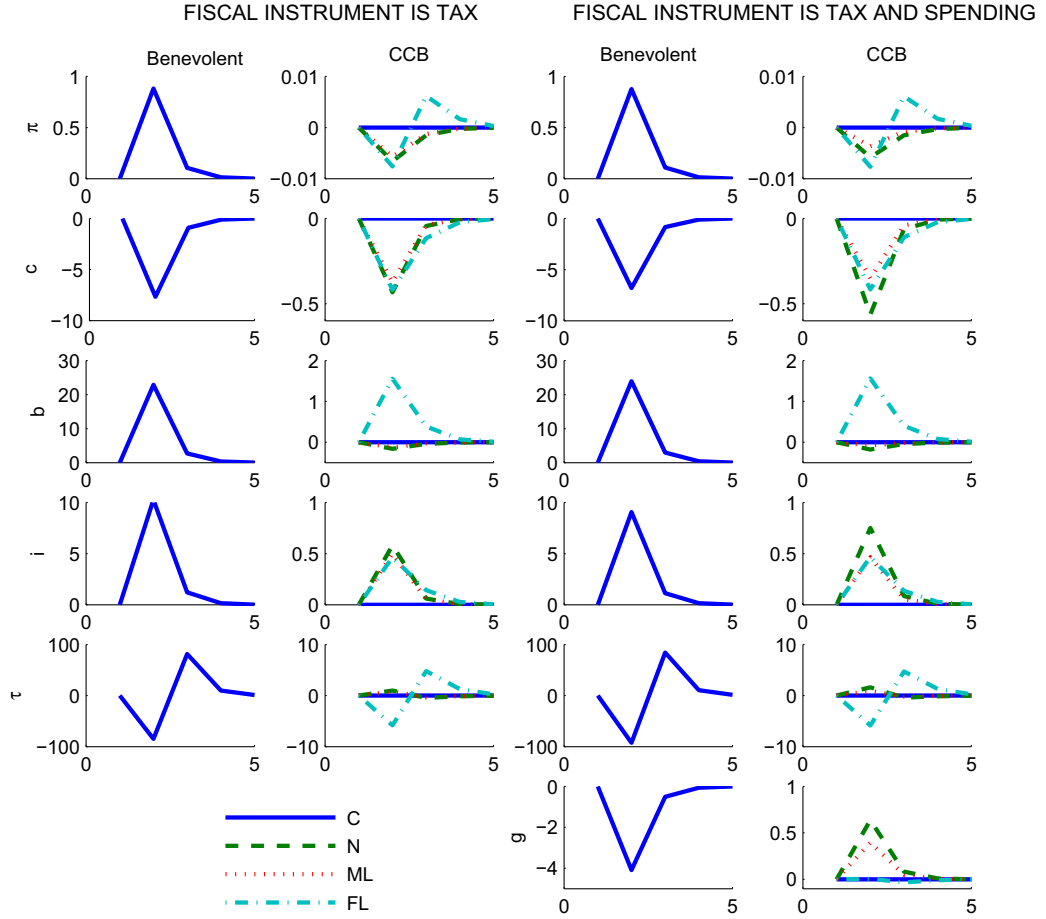


Figure 6: Impulse responses to a unit cost-push shock when fiscal policy uses different instruments. Columns 2 and 4 plot relative responses, relative to the case of cooperation of benevolent authorities. The ‘low debt’ scenario and small degree of conservatism,  $\rho_\pi = 1.06$  are assumed in all cases where applicable.